# Transverse Spin Physics

## Lecture IV

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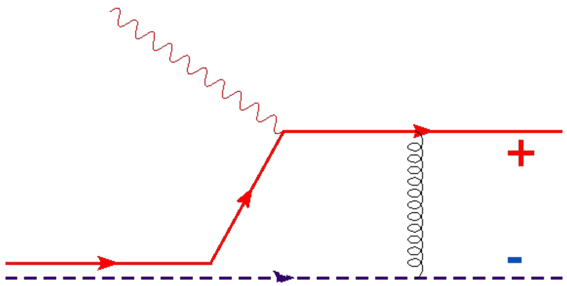
# The plan:

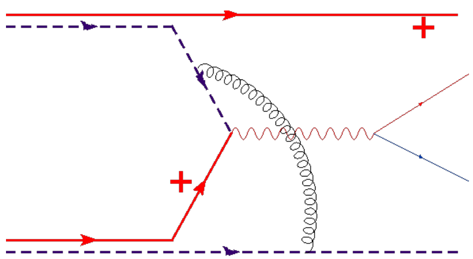
- Lecture I:
  Transverse spin structure of the nucleon
  Overview of past experiments
  History of interpretation
  Overview of present understanding
- Lecture II
  Transverse Momentum Dependent distributions (TMDs)
  Sivers function
  Twist-3
- Lecture III
  Transversity
  Collins Fragmentation Function
  Global analysis
- Lecture IV
  Evolution of TMDs

# Physics of gauge links

Colored objects are surrounded by gluons, profound consequence of gauge invariance.

Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)




Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman, etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond “just” check of TMD factorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou

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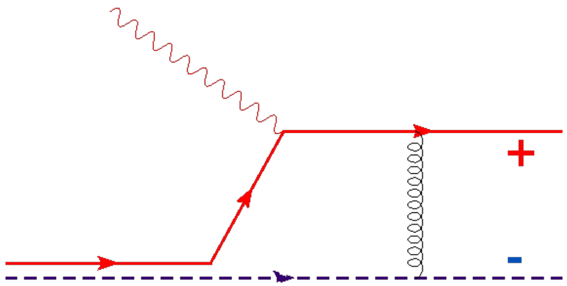


man,

$$f_{1T}^{\perp SIDIS} = -$$

Drell-Yan is at much different resolution scale Q.
EIC will operate at higher Q.
What do we know about evolution of TMDs?

One of the main goals is to verify t... ution.
It goes beyond “just” check of TMD ... ctorization.
Motivates Drell-Yan experiments

AnDY, COMPASS, JPARC, PAX etc
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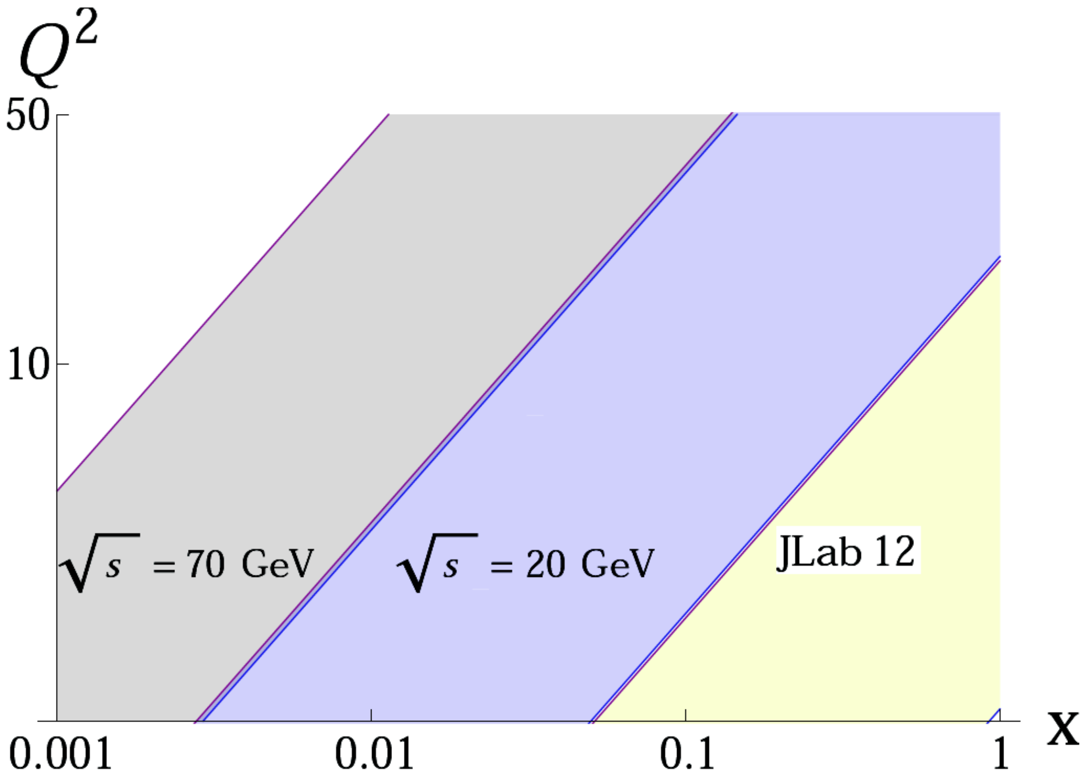
# Kinematics

Kinematics $Q^2 \simeq sxy$

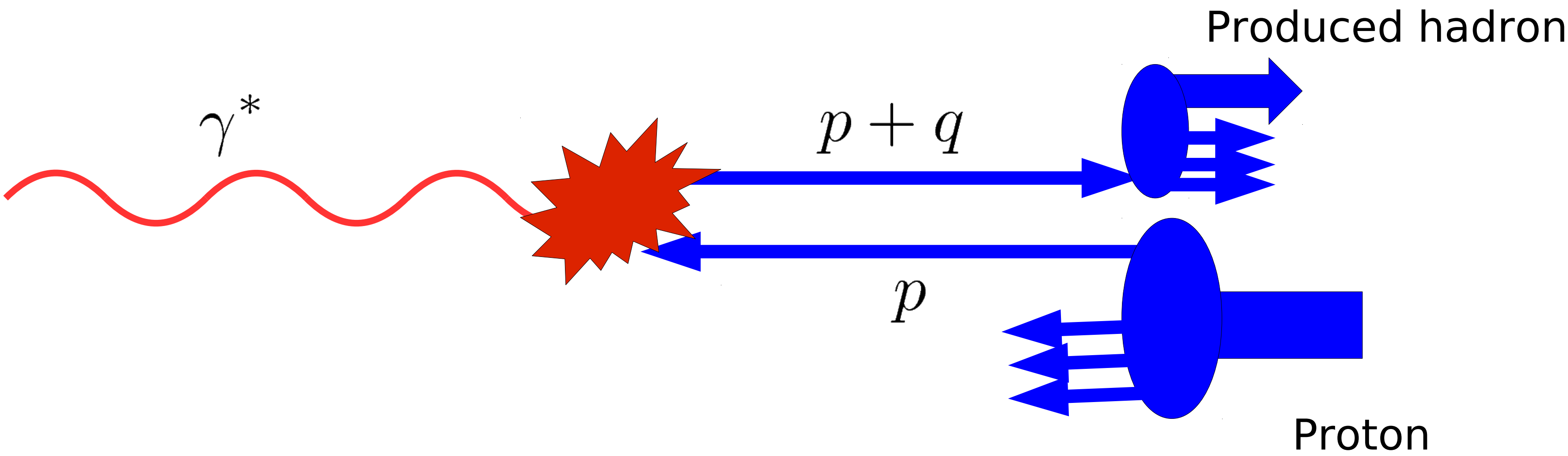



Electron Ion Collider reaches higher Q

Jlab 12 and future Electron Ion Collider are complimentary

# QCD and parton model

Let us calculate SIDIS cross section in parton model:




We work in Infinite Momentum Frame and all partons are collinear to the proton, thus

$$\frac{d\sigma}{dP_T^2} \sim \delta(P_T^2)$$

# QCD and parton model

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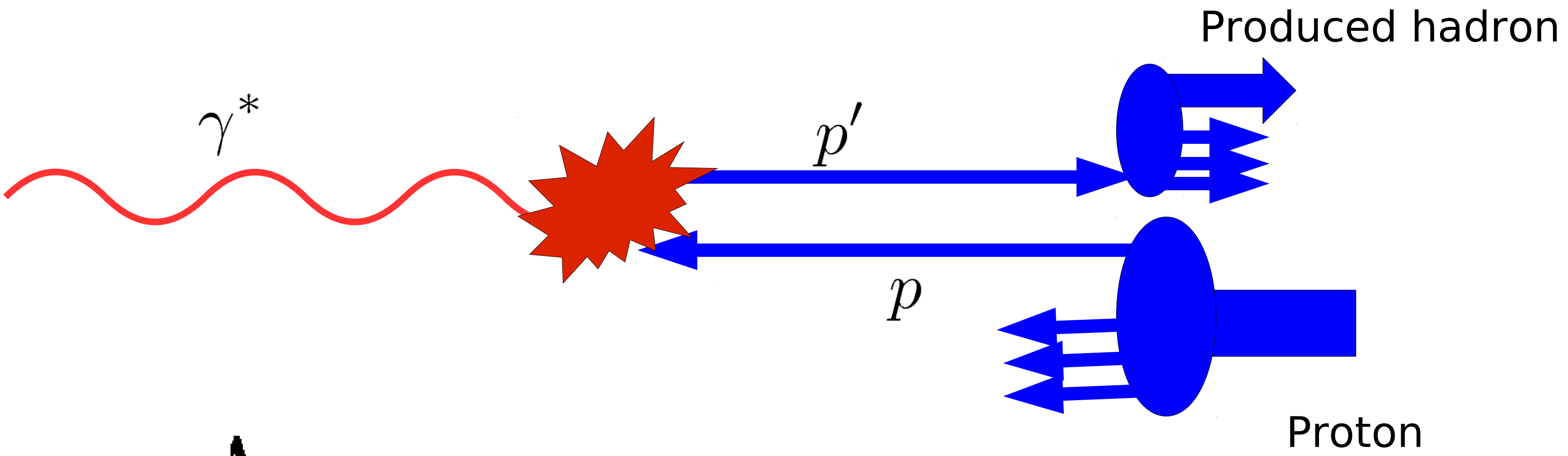
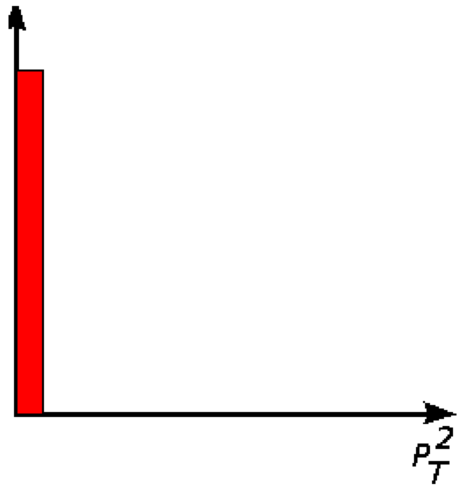



“Theory”



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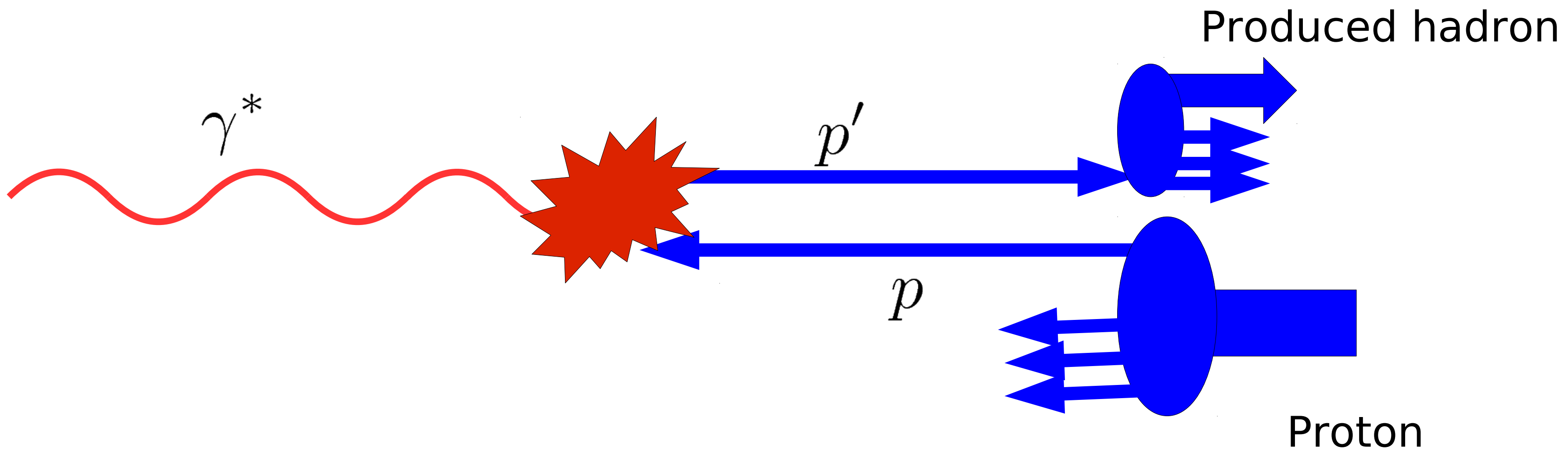


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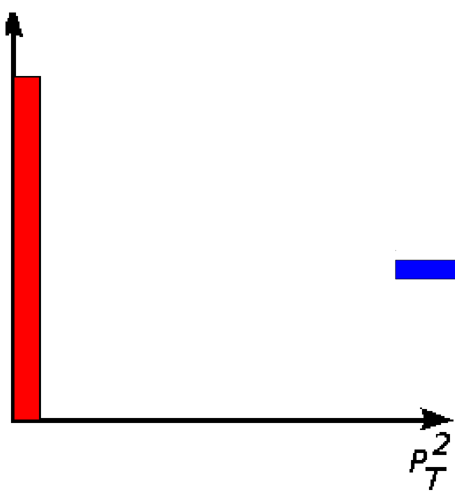
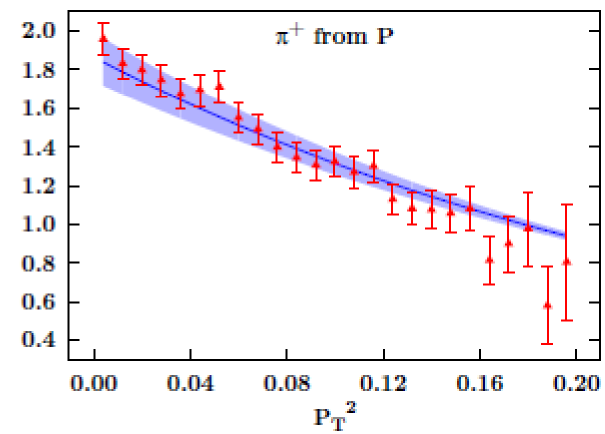
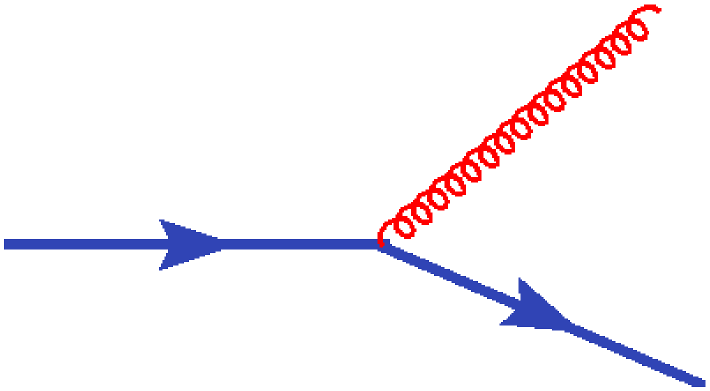



Experiment



"QCD improved" parton model:



Radiation of gluons create transverse momenta

Terms like this appear

$$\left(\alpha_s \ln^2 \frac{Q^2}{P_T^2}\right)^n$$

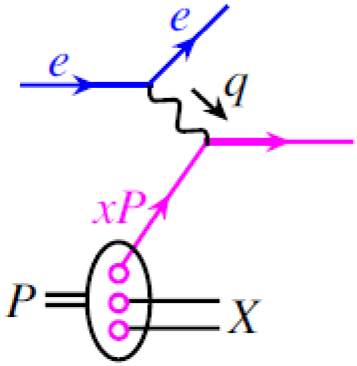
Result is singular as $P_T \longrightarrow 0$ and logs need to be resummed

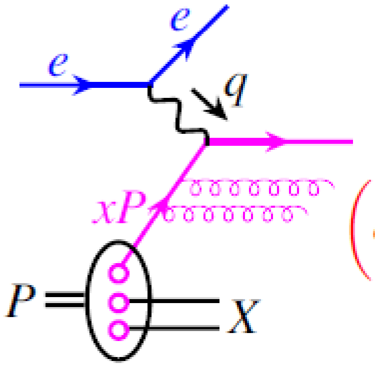
Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Implementation of resummation In QCD

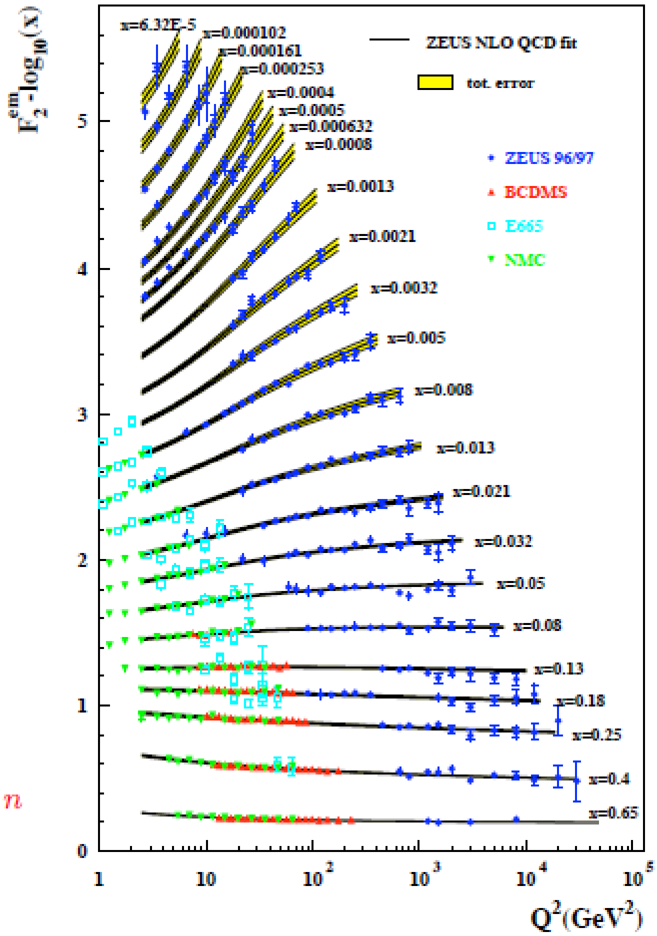
# QCD evolution: meaning

- What is QCD evolution of TMDs anyway?
  - Evolution = include important perturbative corrections
  - One of the well-known examples is the DGLAP evolution of collinear PDFs, which lead to the scaling violation observed in inclusive DIS process
  - What it does is to resum the so-called single logarithms in the higher order perturbative calculations




$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$



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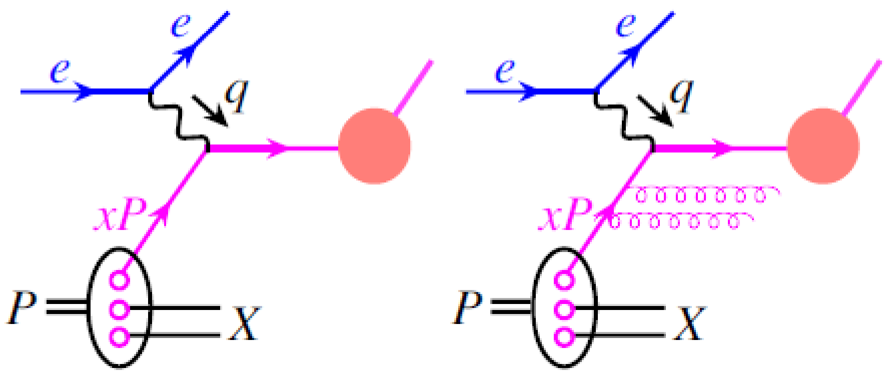
# QCD evolution: TMDs

- TMD factorization works in the situation where there are two observed momenta in the process, such as SIDIS, DY, W/Z production and in the kinematic region where $Q \gg q_T$
- Evolution again = include important perturbative corrections
- What it does is to resum the so-called double logarithms in the higher order perturbative corrections
- For SIDIS: $q_T$ is the transverse momentum of the final-state hadron




$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

# Resummation

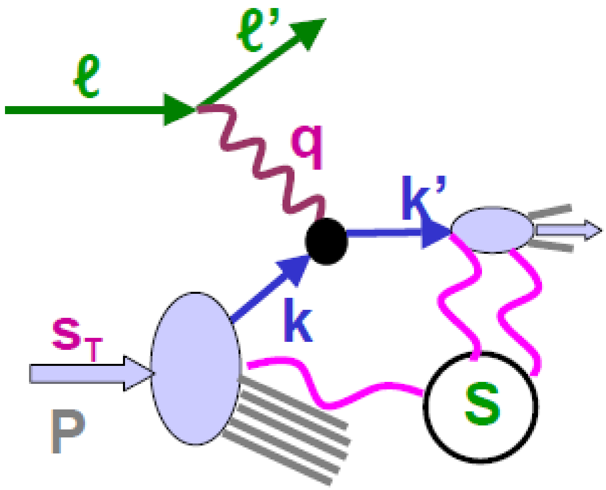
Dokshitzer, Dyakonov, Troyan 1980
Parizi, Petronzio 1979
Collins, Soper 1982
Collins, Soper, Sterman 1985

Resummation (CSS)is in configuration space
Fourier transform is needed for observables

For Drell-Yan

$$\frac{d\sigma}{dq_T} \sim \int d^2 b_T e^{iq_T \cdot b_T} \hat{W}(x_1, x_2, b_T) e^{-S(b_T, Q)} + Y(q_T, Q)$$

Collinear distributions
are contained here



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Large logs (gluon radiation)
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Corrections for large $q_T$

# Resummation

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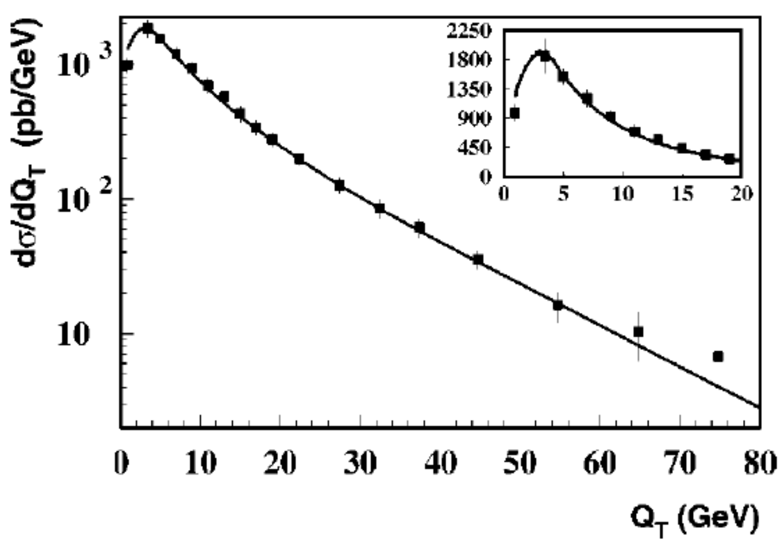
A lot of phenomenology done. Energies from 20 GeV to 2 TeV.

Brock, Landry, Nadolsky, Yuan 2003
Qiu, Zhang 2001



# Evolution of TMDs

One needs a unique definition of TMDs

Foundations of perturbative QCD
Collins 2011

$$W^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F) F_{\bar{f}/P_1}(x_2, \mathbf{k}_{2T}; \mu, \zeta_F)$$
$$\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) + Y(\mathbf{q}_T, Q)$$

$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$ **TMD distribution of partons in hadron**

Rapidity divergence regulator

Renorm group (RG) renormalization

# Evolution of TMDs

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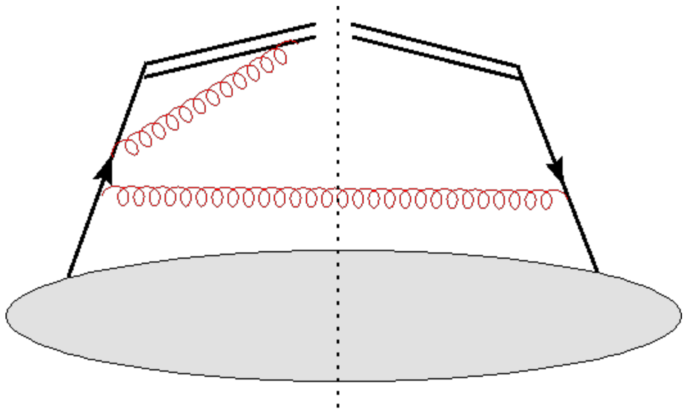

Infinite rapidity of the gluon creates so called rapidity divergence

In collinear PDFs this divergence is canceled between virtual and real gluon diagrams

It is not the case for TMDs
Thus new regulator $\zeta_F$ is needed

$$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu, \zeta_F)$$

Renorm group (RG) renormalization

Rapidity divergence regulator

# Evolution of TMDs

Evolution of TMDs is done in coordinate space $\mathbf{b}_T$

$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T\cdot\mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Why coordinate space?

Convolutions become simple products:

Collins, Soper 1982
Collins, Soper, Sterman 1985
Idilbi, Ji, Ma, Yuan 2004
Boer, Gamberg, Musch, AP 2011

$$\tilde{W}^{\mu\nu} = \sum_f |H_f(Q^2, \mu|^{\mu\nu} \times \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\mathbf{q}_T} \tilde{F}_{f/P_1}(x_1, \mathbf{b}_T; \mu, \zeta_F)\tilde{F}_{\bar{f}/P_1}(x_2, \mathbf{b}_T; \mu, \zeta_F)$$

In principle experimental study of functions in coordinate space
Is possible

Boer, Gamberg, Musch, AP 2011

# Evolution of TMDs

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$$F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_T e^{i\mathbf{k}_T\cdot\mathbf{b}_T} \tilde{F}_{f/P}(x, \mathbf{b}_T; \mu, \zeta_F)$$

Colins Soper 1982
Foundations of perturbative QCD Collins 2011

Complicated in case of Sivers function    Aybat, Collins, Qiu, Rogers 2012

$$F_{f/P^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T; \mu, \zeta_F) = F_{f/P}(x, \mathbf{k}_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, \mathbf{k}_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

Unpolarised part:

$$\tilde{F}_{f/P}(x, b_T; \mu, \zeta_F) = (2\pi) \int_0^\infty dk_T k_T J_0(k_T b_T) F_{f/P}(x, k_T; \mu, \zeta_F)$$

Sivers function:

$$\tilde{F}_{1T}^{\prime\perp f}(x, b_T; \mu, \zeta_F) = -(2\pi) \int_0^\infty dk_T k_T^2 J_1(k_T b_T) F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$

# TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins-Soper kernel in coordinate space

Renormalization group equations

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu), \zeta)$$

# TMD evolution

Energy evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

Collins-Soper kernel in coordinate space

At small $\mathbf{b}_T$ perturbative treatment is possible

TMD:
Collins 2011
Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{K}(b_T, \mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln(\mu^2 b_T^2) - \ln 4 + 2\gamma_E\right) + \mathcal{O}(\alpha_s^2)$$

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

# TMD evolution

Energy evolution

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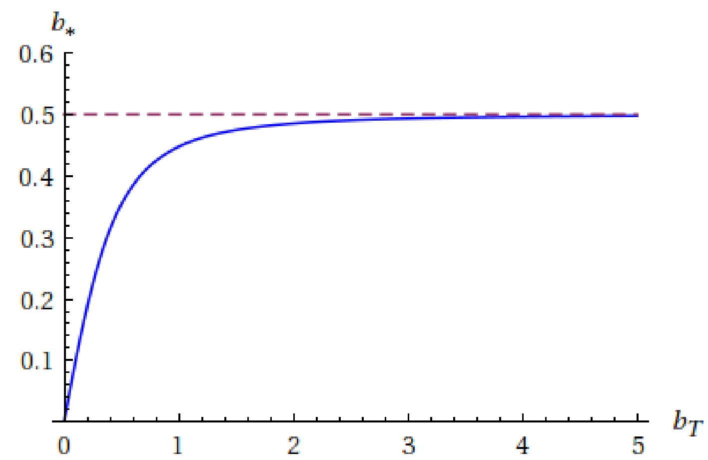
Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$b_*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$$

$$b_{max} = 0.5 \ (\mathrm{GeV}^{-1})$$

Brock, Landry, Nadolsky, Yuan 2003



# TMD evolution

Energy evolution

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Collins-Soper kernel in coordinate space

Large $\mathbf{b}_T$ nonperturbative – matching via $\mathbf{b}_*$ Collins Soper 1982

$$\tilde{K}(b_T, \mu) = \tilde{K}(b_*, \mu) - g_K(b_T)$$

Always perturbative

Non perturbative

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2$$

$$g_2 \simeq 0.68\ (GeV^2)$$

**This function is universal for different *partons* and *processes*!**

Brock, Landry, Nadolsky, Yuan 2003

# TMD evolution

Relation to collinear treatment:

$$\tilde{F}_f(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/f}\left(\frac{x}{\hat{x}}, b_T, \mu, \zeta\right) f_j(\hat{x}, \mu) + \mathcal{O}(\Lambda_{QCD} b_T)$$

Collins Soper 1982

Valid at small $\mathbf{b}_T$ , lowest order:

$$\tilde{C}_{j/f}(\frac{x}{\hat{x}}, b_T, \mu, \zeta) = \delta_{jf}\delta\left(\frac{x}{\hat{x}} - 1\right) + \mathcal{O}(\alpha_s)$$

Higher order for TMD PDFs

Aybat Rogers 2011

Higher order for Sivers function

Kang, Xiao, Yuan 2011

# TMD evolution

Solution

Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = \tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2)$$

$$\times \exp\Big[ - g_K(b_T) \ln \frac{Q}{Q_0} \Big]$$

Non perturbative

$$\times \exp\Big[ \ln \frac{Q}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \Big[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \Big]$$

$$+ \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{Q}{Q_0} \gamma_K(g(\mu')) \Big]$$

Perturbative

Typically for TMDs:

$$\tilde{F}_{f/P}(x, b_T; Q_0, Q_0^2) = F_{f/P}(x; Q_0) \exp\Big( - \frac{\langle k_T^2 \rangle}{4} b_T^2 \Big)$$

# TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Solve evolution equations:

$$\widetilde{f}_1^f(x, b_T; \mu, \zeta_F) = \sum_i \left(\tilde{C}_{f/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu,\zeta_F)} e^{g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}} \hat{f}_{\mathrm{NP}}^q(x, b_T)$$

# TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything at NLO:

$$\tilde{C}_{j'/j}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{\pi}\Bigg\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x) + \\ + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\Bigg\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{\pi}\Bigg\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x) + \\ + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\Bigg\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, \boldsymbol{b}_T; \mu; \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_F}{\pi}\Bigg\{ \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\left(\frac{2x}{1-x}\right)_+ + \\ + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_E}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\Bigg\} + \mathcal{O}(\alpha_s^2)\,.$$

# TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

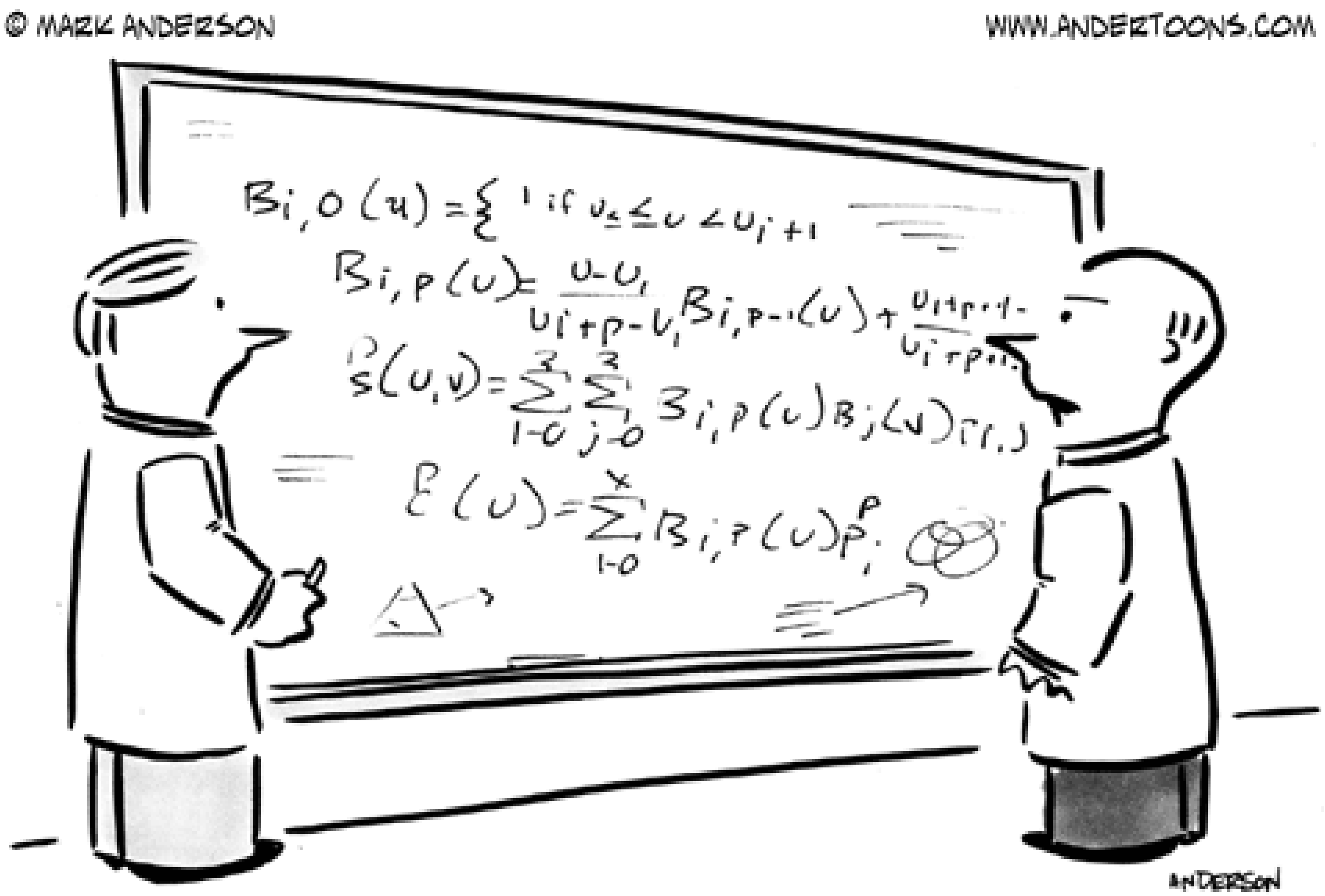
Simplify:

$$\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}\,(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}\,(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \mathcal{O}(\alpha_s^2).$$

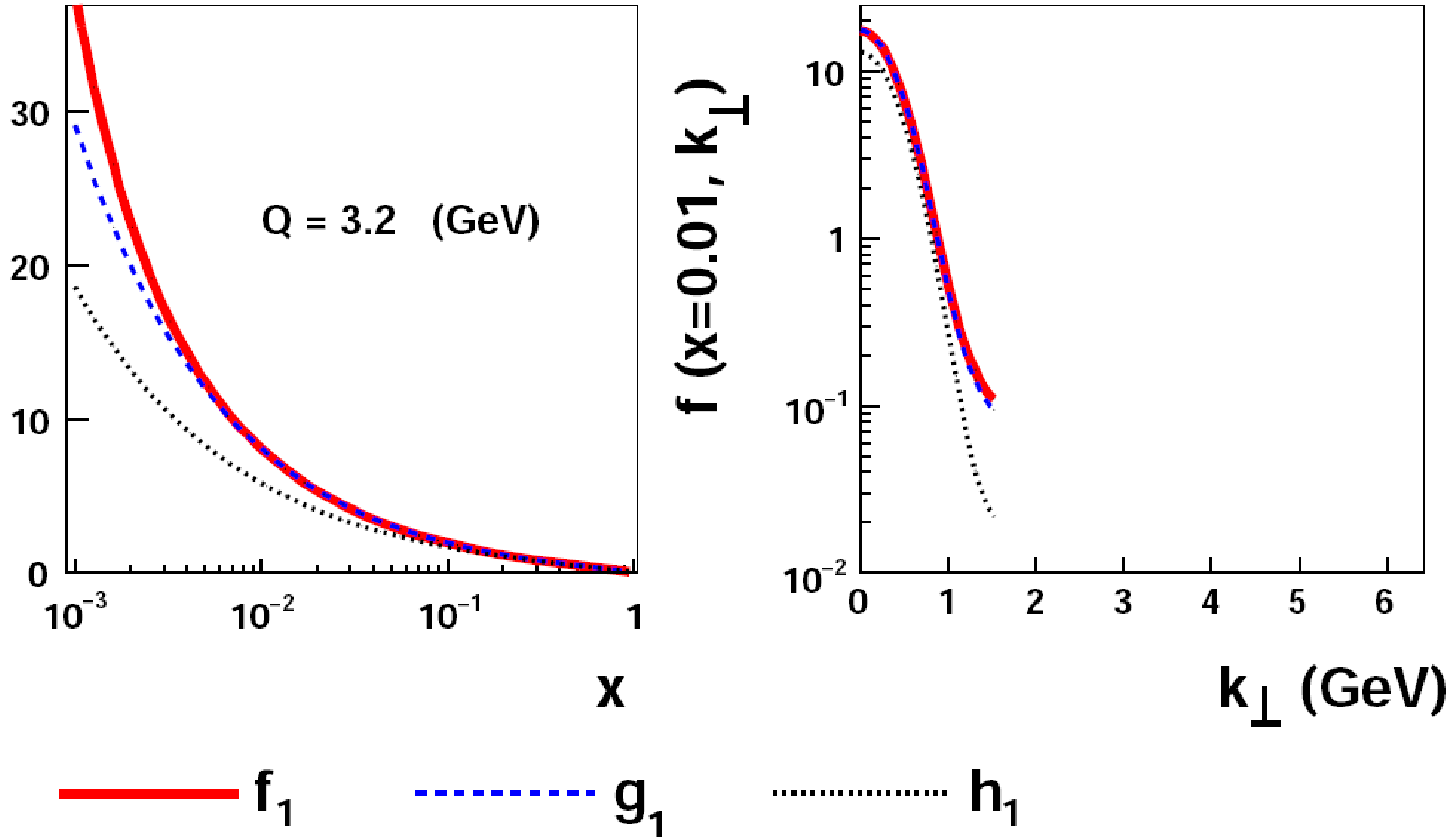
# What does it mean?




"What the hell is *that* supposed to mean?!"

# Phenomenology

A. Bacchetta, AP, 2013



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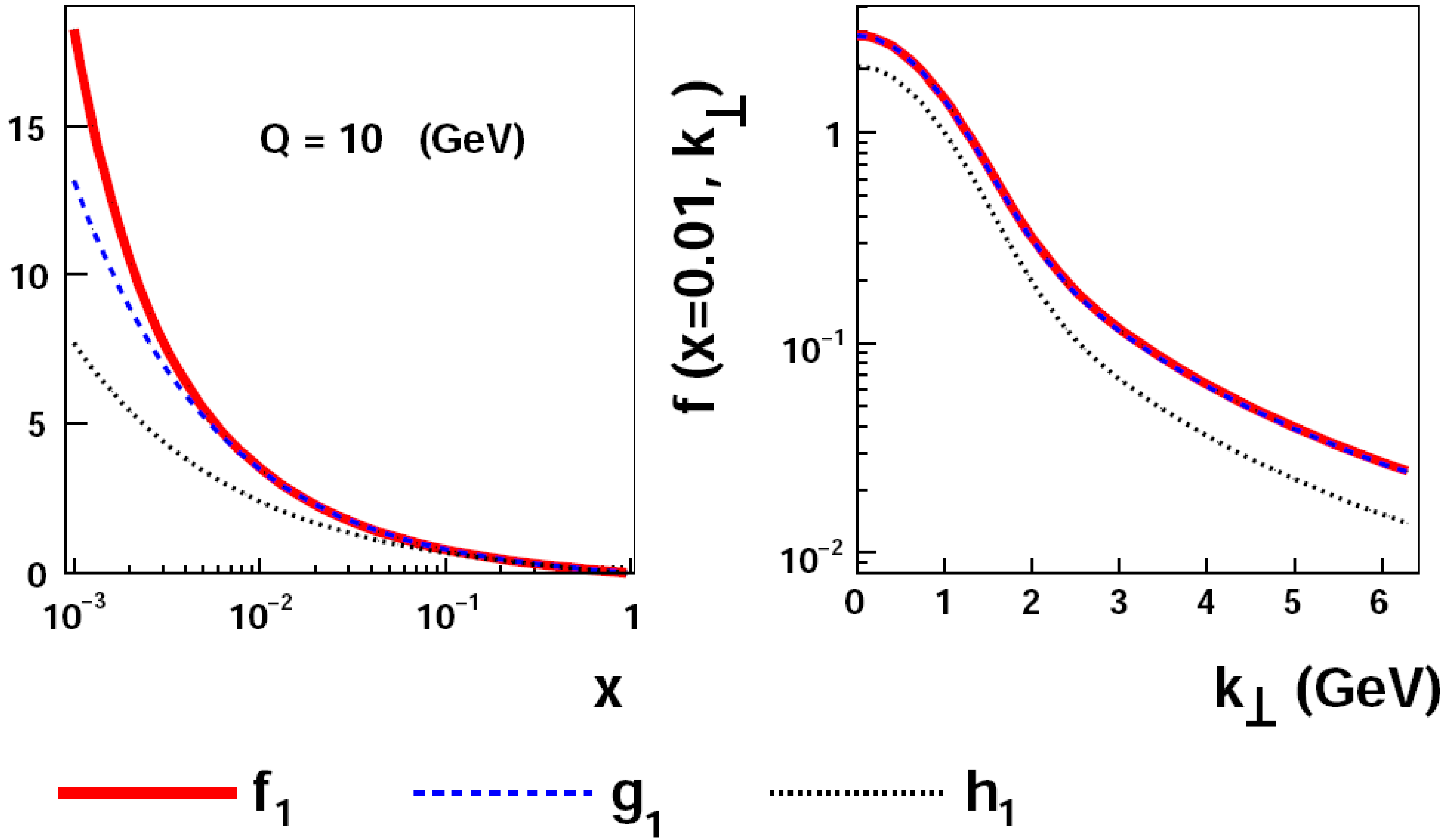

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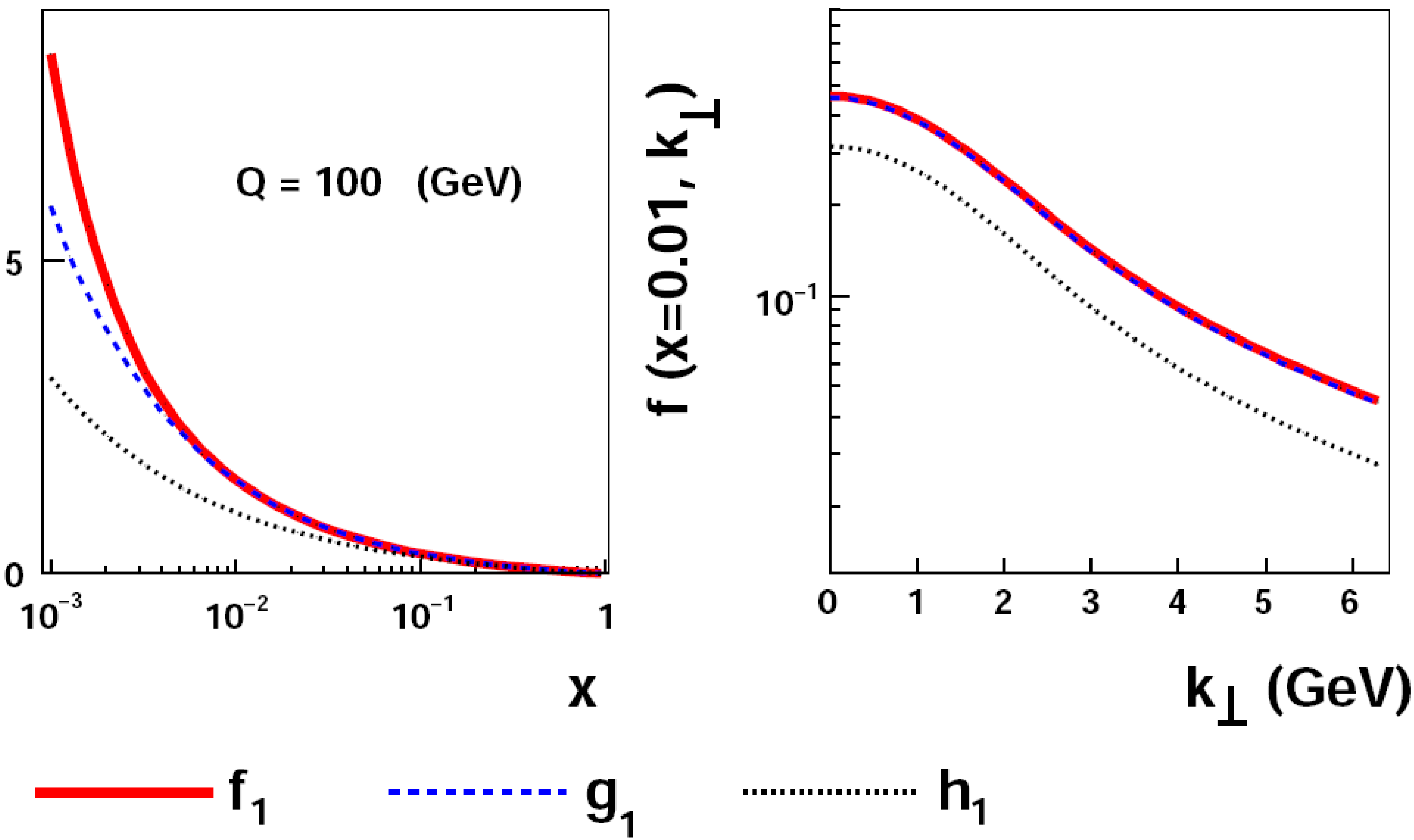
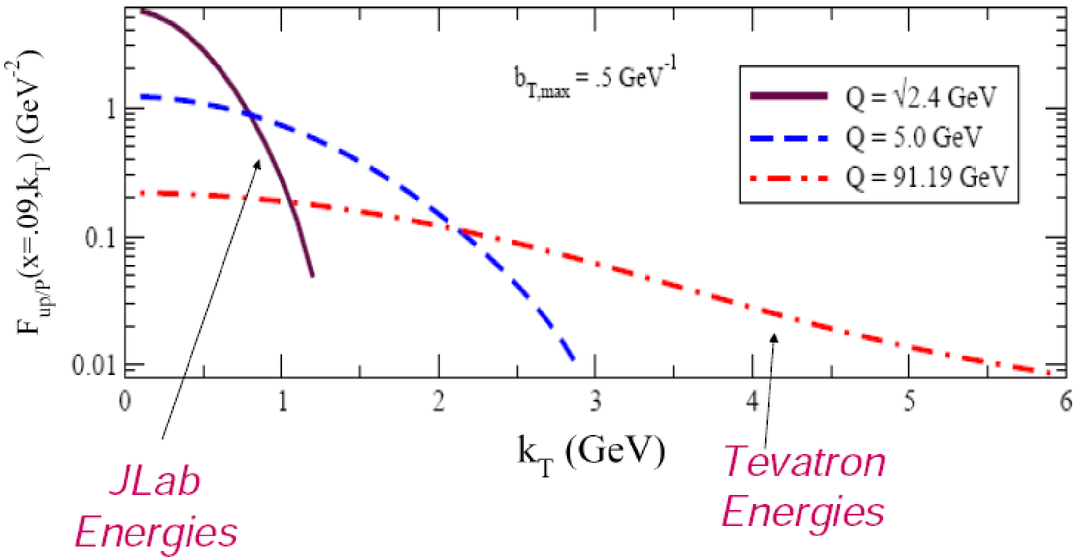


# TMD evolution

Solution

Rogers, Aybat 2011
Aybat, Collins, Qiu, Rogers 2011

$$\tilde{F}_{f/P}(x, b_T; Q, \zeta_F) = F_{f/P}(x; Q_0) \exp\left( - \underbrace{\left[ \frac{\langle k_T^2 \rangle}{4} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right]}_{\text{Non perturbative}} b_T^2 \right)$$



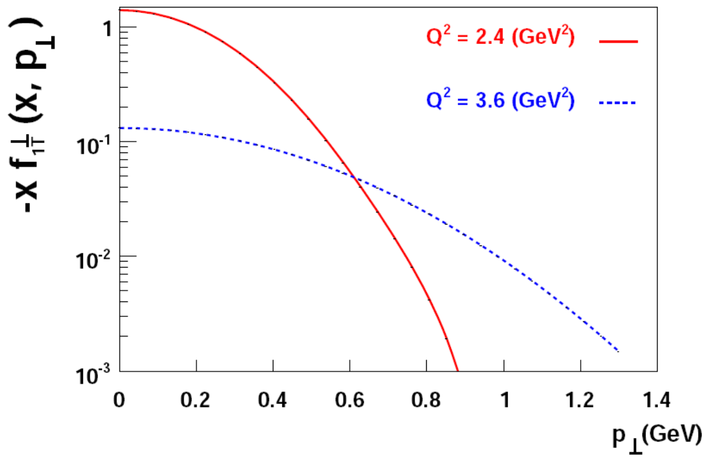

Gaussian behaviour is appropriate only in a limited range

TMDs change with energy and resolution scale

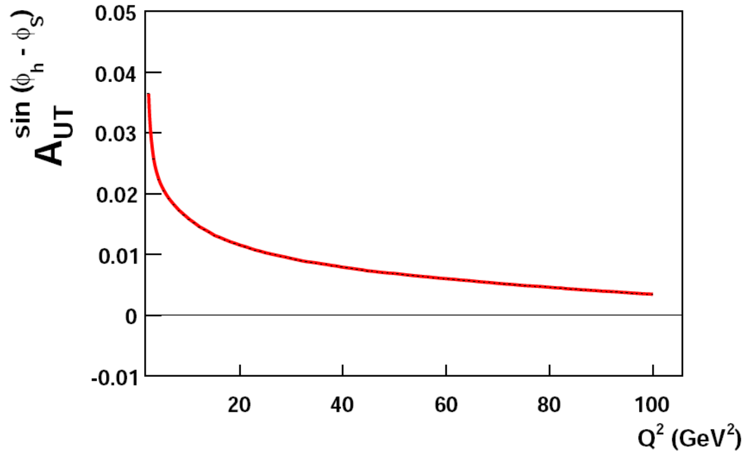
# TMD evolution

This is the first implementation of TMD evolution for observables

Aybat, AP, Rogers 2011




Asymmetry changes with $Q^2$



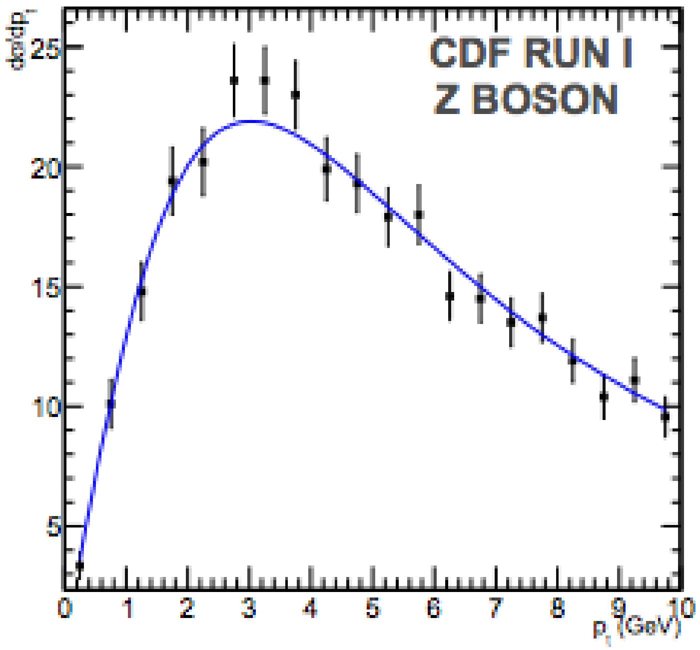

Functions change with energy

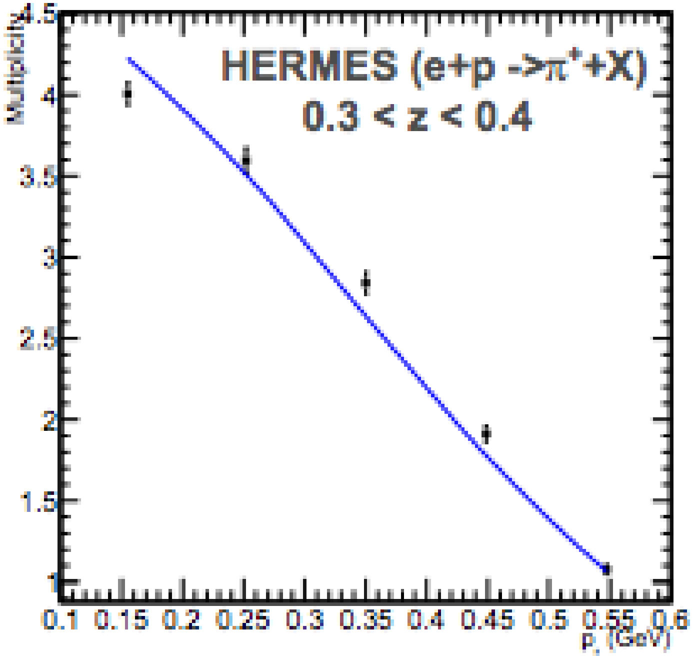
Phenomenological analysis with evolution is now possible

# Could we do better?

- A new fit for DY seems also describe SIDIS data

Sun-Isaacson-Yuan-Yuan,1406.3073




$$\exp\left(-g_2 b^2 \ln Q^2 + \cdots\right) \qquad \exp\left(-g_2 \ln b \ln Q^2 + \cdots\right)$$

Non perturbative function is modified

- TMD evolution kernel is NOT entirely perturbative (collinear evolution kernel is purely perturbative)
- We have a TMD distribution F(x, kt; Q) measured at a scale Q
  - It is easy to deal in the Fourier transformed space

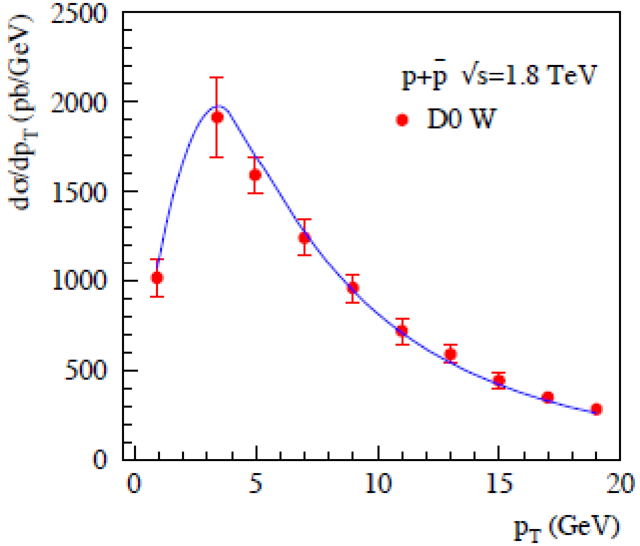
$$F(x, b; Q) = \int d^2k_\perp e^{-ik_\perp \cdot b} F(x, k_\perp; Q)$$

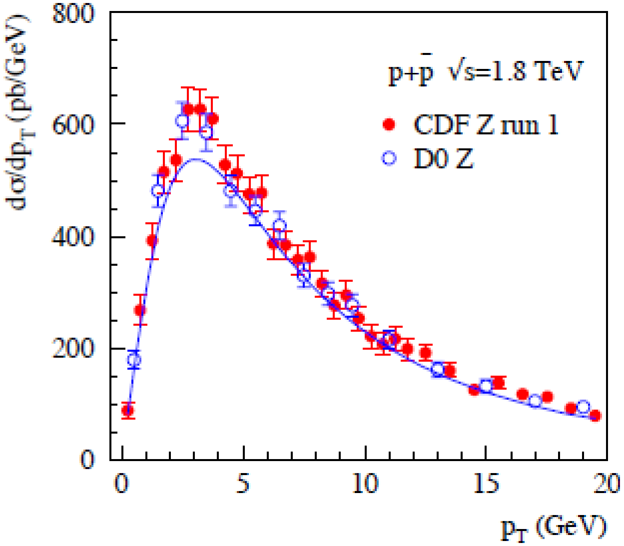
  - Perturbatively it evolves from an initial scale $c = 2e^{-\gamma_E} \sim O(1)$

$$F(x, b; Q) = F(x, b; c/b) \exp\left\{-\int_{c/b}^{Q} \frac{d\mu}{\mu}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\}$$

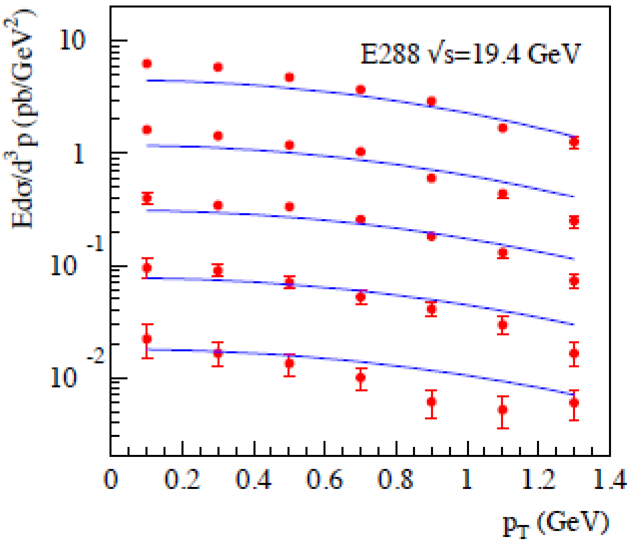
$$A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n, \qquad B = \sum_{n=1} B^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

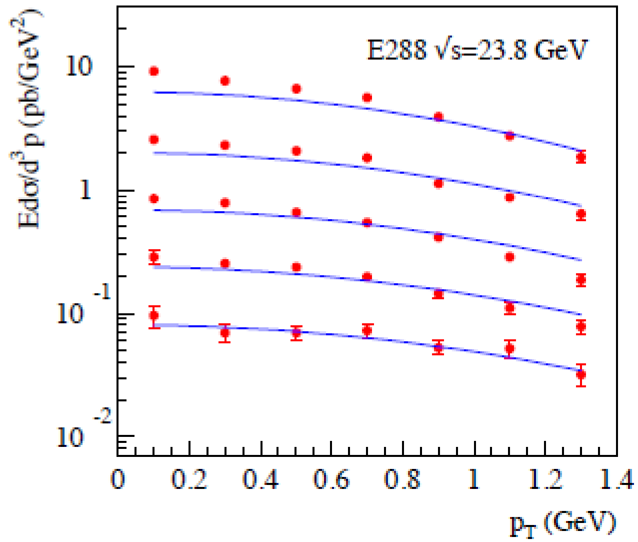
- Description of W/Z data at Tevatron and LHC: not a fit, but a reasonable “tune”



# Drell-Yan lepton pair production

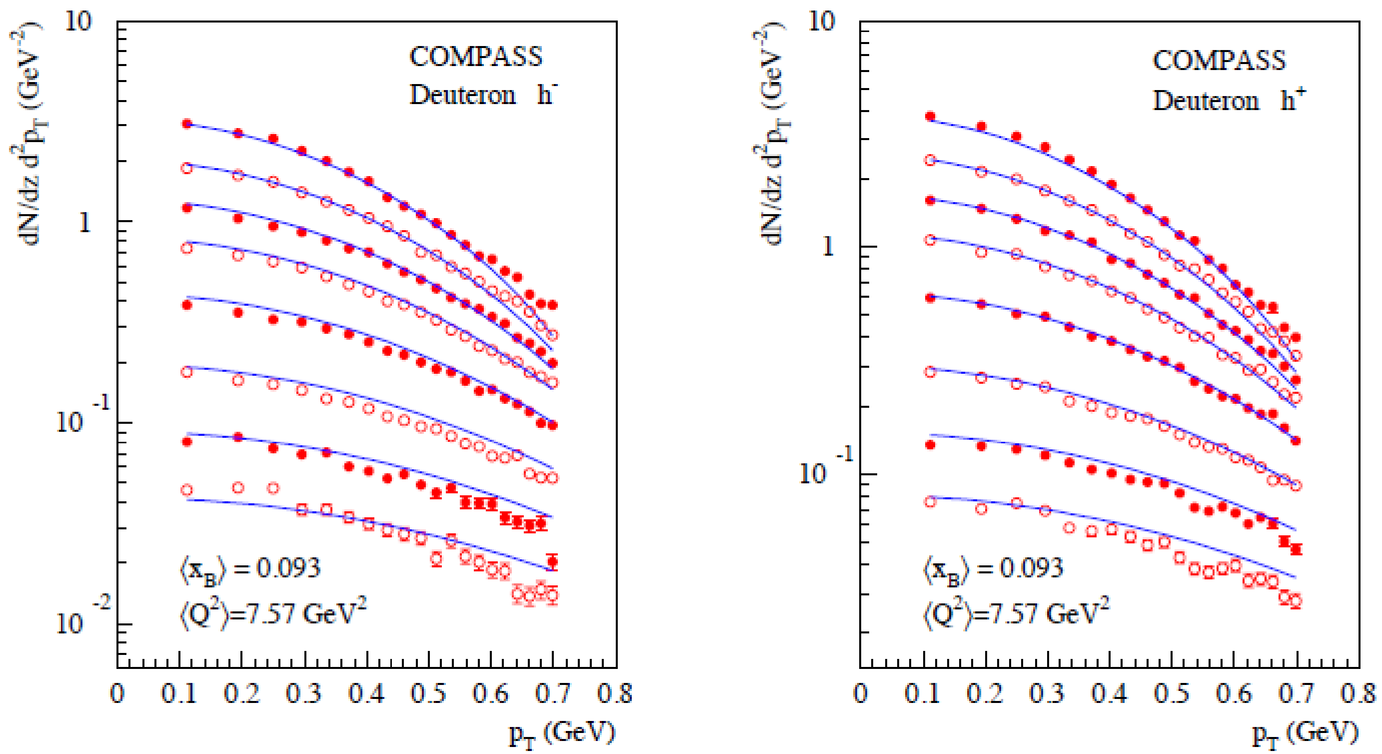


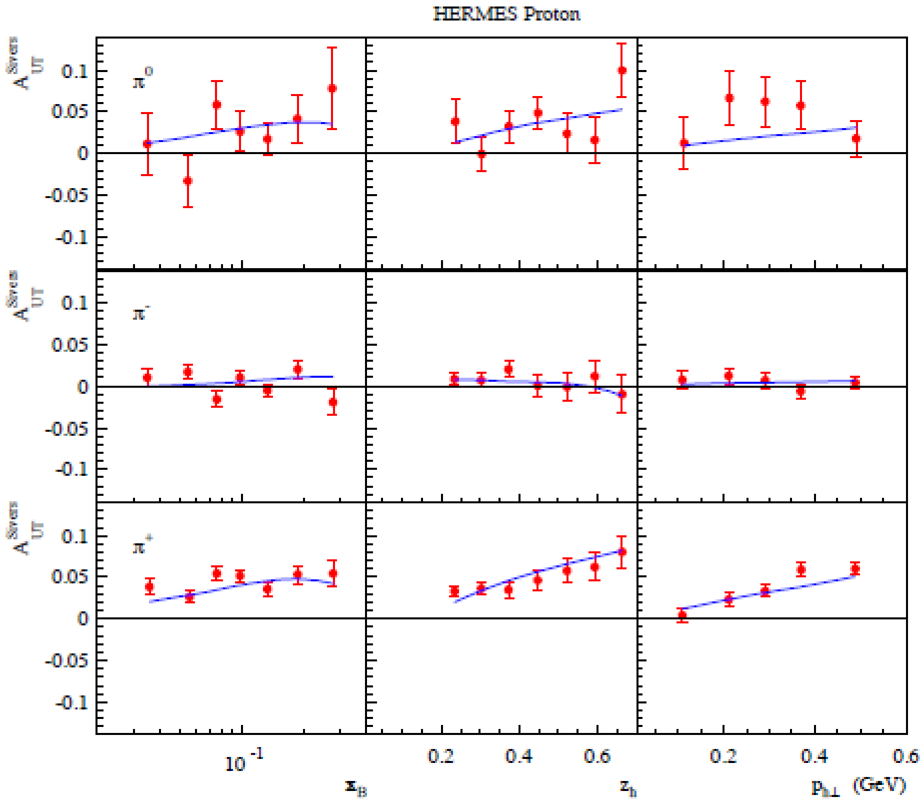
Echevarria-Idilbi-Kang-Vitev, 1401.5078

# Multiplicity distribution in SIDIS 1

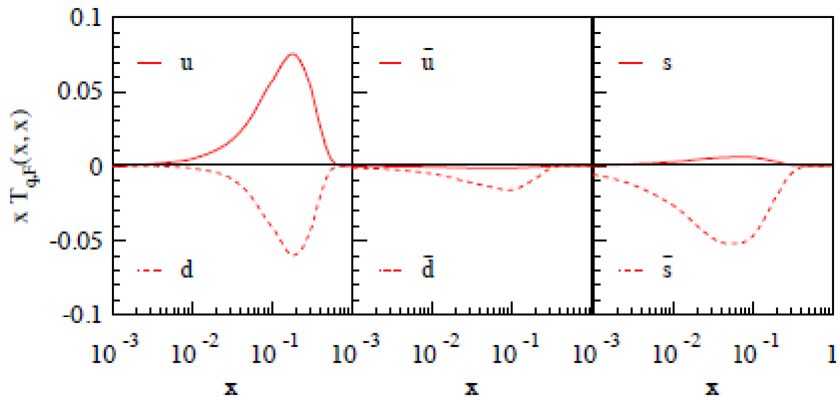
- Comparison with COMPASS data



- Once the non-perturbative part is constrained, use the same formalism to describe the Sivers effect

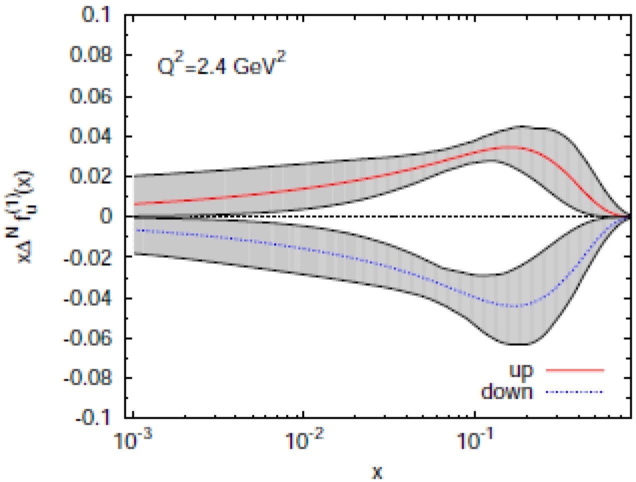


- Sivers function to be compared to that without evolution




Echevarria-Idilbi-Kang-Vitev 14

With TMD evolution




Anselmino 14

No TMD evolution

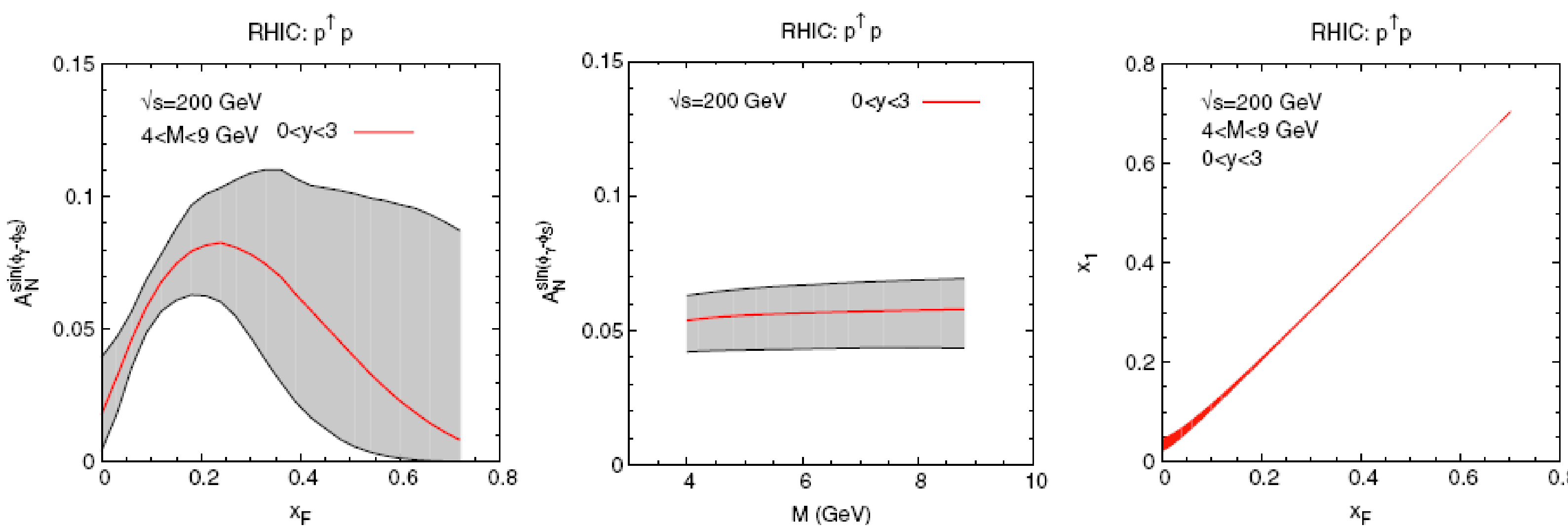
# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^q(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

Analysis at LO in hadronic cm frame
Anselmino et al (2009)

$$x_1 = \frac{x_F + \sqrt{x_F^2 + 4M^2/s}}{2} \approx x_F$$

In DY we probe Sivers function at $x_F$
Anselmino et al (2009)



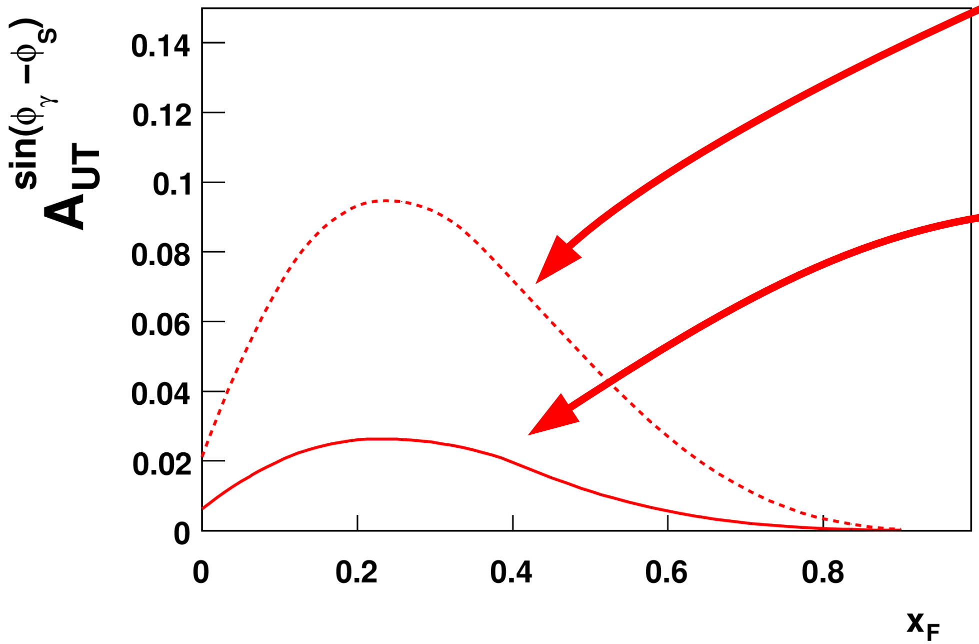
# Drell Yan

$$A_N = \frac{\sum_q f_{1T}^{\perp q}(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}{\sum_q f_1^q(x_1, p_T) \otimes f_1^{\bar{q}}(x_1, p_T)\sigma_{q\bar{q}}}$$

**Analysis in hadronic cm frame**

No TMD evolution

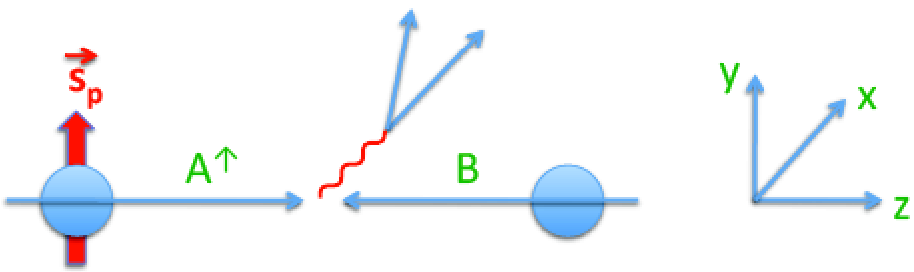
With TMD evolution

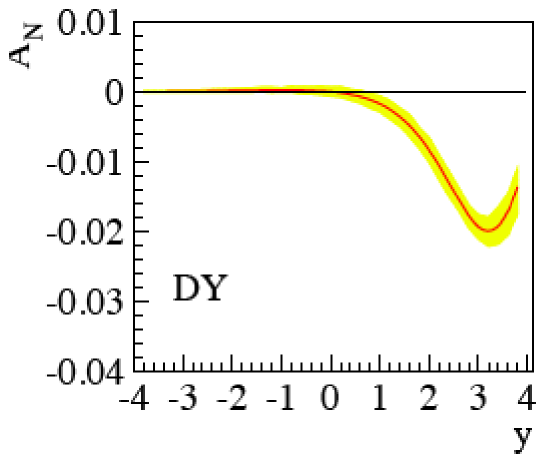


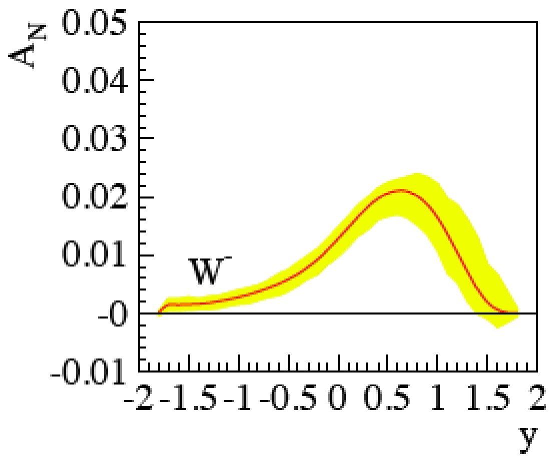

Asymmetry is suppressed with respect to LO analysis

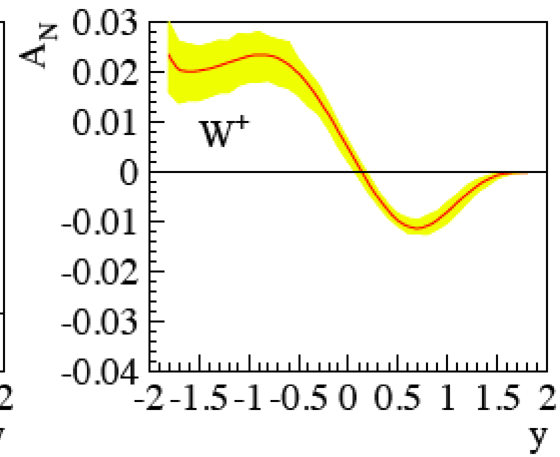
# Predictions for DY, W/Z

- At 510 GeV RHIC energy



# Evolution

- Theory formulated in 2011
- A lot of work to do
- Preliminary results are very promising